A STUDY OF THE GENERATION OF LOW FREQUENCY RANDOM NOISE

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SUMMARY

It was the purpose of this investigation to design, construct, and test a low frequency random noise generator to be used with an analog computer. The generator was built to have an output with a constant mean value, a Gaussian amplitude distribution, and a power spectral density that is flat within 1 db to 50 cps.

The problem is handled in the following manner: A gas filled thyratron is used as the noise source. An audio amplifier is constructed to amplify the noise to a usable level. Here part of the noise is rectified and the average value of the rectified noise is fed back to a remote cutoff pentode in the amplifier to serve as an automatic gain control. The rest of the noise output is transformer coupled into a balanced ring modulator using a square wave for switching. The output of the modulator is a spectrum of the sum and difference frequencies of the noise and the square wave frequency. By directly coupling a low pass filter with cutoff at approximately 70 cps to the output of the modulator, a noise output with a flat spectrum from 0 to at least 50 cps can be obtained. Again amplification is necessary; therefore, a d-c amplifier is used to obtain a usable output.

The tests which were made on the noise output of the generator are: plotting of the amplitude distribution, determination of the autocorrelation function, computing of the average and rms values, and investigation of the power spectral density. The results of these tests show that the generator provides a noise output with the desired characteristics.

CHAPTER I

INTRODUCTION

In all electronic equipment there are certain types of noise which even theoretically cannot be eliminated. This inherent noise arises from the atomic nature of matter and electricity. Noise of this nature is a typical example of a random (or stochastic) process, and therefore can only be investigated by using the notion of probability. These different noise sources which cannot be eliminated produce what is usually termed fluctuation noise. The predominant sources of fluctuation noise are thermal effect, shot effect, and magnetic fluctuation effect.

The response of linear systems to random noise can be determined by analytical techniques. However, there is at present no general analytical method for the treatment of a random signal which is either transmitted through a nonlinear system or undesirable noise which is present in the nonlinear system. However, by using a noise generator in connection with an analog computer, many noise and random signal problems which occur in nonlinear systems can be simulated. Through these simulated studies the response of nonlinear systems to noise can be determined.

It was the purpose of this research project to design, construct, and test a noise generator to be used with an analog computer. This generator has a random noise output with a constant average value. The amplitude distribution of the noise is Gaussian. Caution was taken to insure that no periodic fluctuations will appear at the output of the generator. The frequency spectrum is virtually flat from 0 to about 50 cycles per second and then drops off rapidly.

Many studies have been made of the sources of noise in electronic systems and in the atmosphere. Possible sources of noise of different natures are treated extensively by Goldman (1), physical sources of noise in electronic equipment are discussed by J. R. Pierce (2), and the nature of atmospheric noise is explained in many articles. The most significant sources of noise which are presently being used in noise generators are: gas diodes, gas diodes in magnetic fields, resistors, crystals, arc discharge devices, photomultiplier tubes, and radioactive sources.

The frequency spectrum and power output of gas diodes make them a feasible source of noise in the design of low frequency noise generators. The tube generates noise because of the random fluctuations in positive ion layers around the cathode and the random collisions made between electrons and gas molecules as electrons are drawn to the plate. Because of these random processes the noise output of the tube is random and has very nearly a normal amplitude distribution. Tests made by J. D. Shaffer (3) show that a 2D21 thyratron operating as a diode serves as very reliable noise source. Also, in <u>Military Specifications for Electron Tubes</u> (4) a 6D4 is shown to be a good source of noise.

In much of the present literature the basic design consists of the following stages: a noise source, a stage of tuned amplification, a detector (usually of the synchronous type), and a low pass filter. Many refinements on this basic system have been used to improve its performance. Automatic gain control has been used to give a more stable output (5). DC-AC choppers have been employed as detectors to eliminate the d-c drift inherent in many electronic detectors (5). However, the use of a chopper greatly limits the center frequency which can be used in the tuned amplifier. One of the most recent designs uses as a detector two peak rectifiers which have outputs that are Rayleigh distributed. These two Rayleigh distributions can be combined to give a distribution which is almost Gaussian (6).

The Goodyear Aircraft Corporation has constructed a low frequency noise generator employing an entirely different technique (7). In this generator a self-quenching Geiger-Mueller tube with associated circuitry is used to produce current pulses when particles above a certain energy threshold pass through the tube. These pulses are amplified and used to trigger a bistable multivibrator producing a square wave of constant amplitude and random time crossing. This output can be filtered to obtain different frequency and amplitude distributions.

For this research project the problem of building a low frequency random noise generator is handled in the following manner. A gas filled thyratron is used as the noise source. An audio amplifier is constructed to amplify the noise to a usable level. Here a fraction of the noise is rectified and the average value of the rectified noise is fed back to a remote cutoff pentode in the amplifier to serve as an automatic gain control. The remainder of the noise output is transformer coupled into a balanced ring modulator using a square wave for switching. The output of the modulator is a spectrum of the sum and difference frequencies of the noise and the square wave frequency. By directly coupling a low pass filter with cutoff at approximately 70 cps to the output of the modulator, a noise output with a flat spectrum from 0 to at least 50 cps can be obtained. Again amplification is necessary; therefore, a d-c amplifier is used to obtain a usable output.

The tests of random signals, such as noise, which seem to be necessary are: plotting of the amplitude distribution, determination of the autocorrelation function, computing of the average and rms values, and investigation of the power spectrum. All of these tests have been performed on the noise generator and are recorded in the following chapters.

CHAPTER II

DESIGN CONSIDERATIONS

Introduction--The primary object of this research project is to design and construct a low frequency random noise generator. The basic block diagram of this generator is shown in Figure 1. The schematic diagram of the noise generator is given in Figure 9 in Appendix B.

Briefly, the system is designed to create a primary source of wide-band noise, amplify the audio range of this noise, heterodyne the noise output of the audio amplifier with a balanced modulator, and filter and amplify the output of the modulator to obtain a usable low frequency noise output. In this chapter the design and characteristics of each section of the generator will be discussed, and the relation of each section to the entire system will be explained.

Primary Noise Source--A type 6D4 thyratron in a magnetic field is used as the primary source of noise. The thyratron is operated as a diode by connecting the grid and the cathode to ground and connecting the plate to a +300 volt supply through a variable resistance. By means of a variable resistance in the plate load of the tube, the optimum value of plate resistance can be obtained for any 6D4 which is used in the circuit. It has been found that some tubes used as a noise source would operate as relaxation oscillators at a frequency in the order of 100 kc. By adjusting the plate resistance, these oscillations could be suppressed.

The 6D4 thyratron, like the 2D21 and other grid-controlled, gas discharge tubes, produces noise from the random fluctuations in the dense

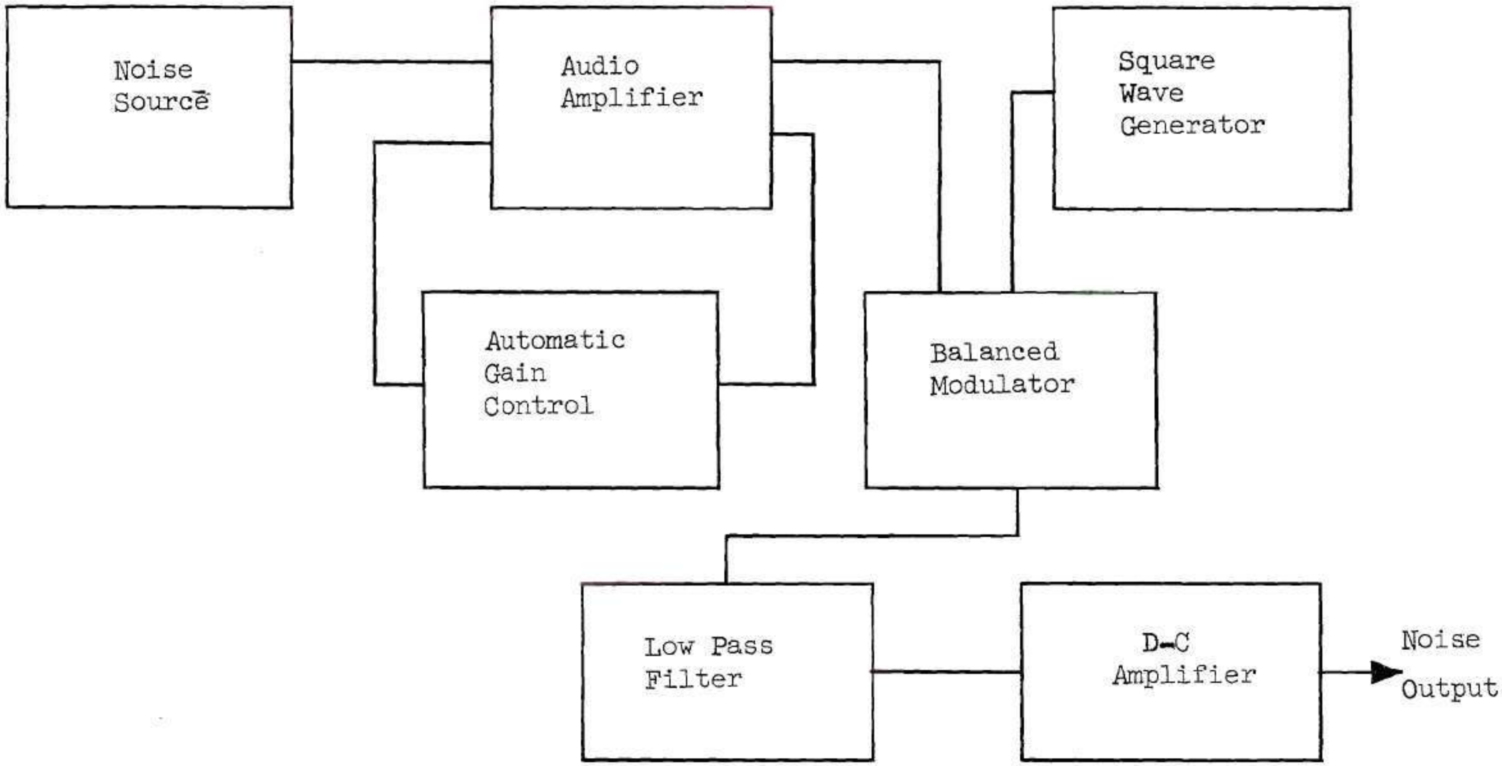

Figure 1. Block Diagram of Noise Generator.

layer of positive ions near the cathode. This noise has been shown to have a normal probability density distribution and the upper limit of the frequency bandwidth of this noise varies inversely with the atomic weight of the gas (8). In the case of a 6D4 which is filled with argon with an atomic weight of 39.944 there is noise with frequency components up to 1 megacycle which is more than sufficient for use in this generator. Ions can be produced almost instantaneously, but have a finite life inversely proportional to their mobility. The mobility varies inversely with atomic weight; therefore, the reciprocal of the lifetime determines the upper frequency limit.

In order to suppress sharp resonance peaks associated with natural oscillation frequencies of the ions in the 6D4, a transverse magnetic field from a permanent magnet was applied across the electric field in the tube. This magnetic field not only suppresses resonant peaks, but also increases the high frequency noise output from the tube (9).

The magnetic field causes the low energy electrons emitted from the cathode to be turned back to cathode. These electrons would otherwise go directly to the anode without colliding with gas atoms since the gas becomes almost transparent to low energy electrons. Therefore, the removal from the anode current of these low energy electrons which would have been velocity-modulated by ionic oscillations breaks up the sharp resonance condition (9).

When the circuit and magnet are adjusted properly, the 6D4 gives a noise output with frequency components up to 5 megacycles. This output then serves as the primary source of noise and is put into an audio amplifier with automatic gain control to obtain a usable amplitude of noise with a stable average value.

Audio Amplifier with A.G.C.--A variable gain amplifier with a frequency response of from 30 cps to 20 kc was designed. A remote cutoff pentode was used to obtain variable gain and after two more stages of amplification a cathode follower output is provided. The circuit diagram for this amplifier is given in Figure 9 in Appendix B.

The first stage of amplification is a remote cutoff pentode (6BA6) whose cathode is maintained at a constant voltage by connecting an OA3 voltage regulator tube in the cathode circuit. Also in the cathode circuit is a d-c milliammeter to indicate the amount of plate current in the 6BA6. By feeding to the control grid a negative voltage whose magnitude is proportional to the average amplitude of noise out of the amplifier, the gain of the 6BA6 can be controlled. Since the bias on the 6BA6 determines the amount of plate current the tube will draw, the bias can be determined by reading the plate current on the d-c milliammeter in the cathode circuit. A plot of bias voltage versus plate current for the pentode is shown in Figure 14 in Appendix B. Therefore, the bias can be set and monitored without having to connect a meter between the control grid and cathode of the 6BA6. The rest of this stage of amplification and the following amplifier stages are of conventional design.

The cathode follower stage, which serves as the output of the amplifier, transformer couples part of the noise into a balanced ring type modulator, while the other part of the noise is coupled back into a circuit which provides the bias for the 6BA6 to obtain automatic gain control. The AGC circuit rectifies the noise and averages the rectified noise using a high gain d-c amplifier connected to obtain a transfer function of

$$\frac{E_{out}}{E_{in}} = \frac{1}{\sqrt{1 + .25\omega^2}} . \tag{1}$$

This d-c amplifier was designed at the Georgia Tech Engineering Experiment Station. The schematic diagram for this amplifier is shown in Figure 13 in Appendix B. Since an identical amplifier is used as an output amplifier, the operation of these amplifiers will be covered in the discussion of the output stage of the generator.

After the rectified noise has been averaged with the time constant of .5 seconds, this slowly varying voltage is superimposed on a d-c supply* and applied through a 1 megohm resistance to the control grid of the 6BA6. By adjusting the voltage of the d-c supply, the bias on the 6BA6 may be set to any desired voltage level. The bias will then vary about the set voltage level causing the gain of the tube to vary inversely with an increase or decrease in amplitude of the noise. Thus automatic gain control is achieved.

Square Wave Generator--The switching for the ring modulator is provided by a 100 volt peak to peak square wave with a frequency of about 3.5 kc. Since the noise after amplification has frequency components extending from 30 cps to 20 kc, it is only necessary for the frequency of the switching waveform to fall well within the band of the amplifier. Slight

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*This d-c supply consists of a 90 volt battery connected across a .5 megohm potentiometer.

variations in the switching frequency over a period of time should not affect the output of the generator.

The square wave is derived by clamping the output of a stable frequency, free-running multivibrator to +50 volts and -50 volts. Two 6AK5 tubes are connected as a frequency stable multivibrator (10). Without clamping a peak to peak voltage of 250 volts can be obtained with a plate supply voltage of 300 volts. A potentiometer is provided in the timing circuit of the multivibrator so that through adjustment a symmetrical square wave can be obtained.

The clamping circuit for the square wave is of conventional design using the twin diodes of a 6AL5 tube. After clamping, the rise time of the square wave is less than 10 μsec. The square wave is then put through a cathode follower and transformer coupled into the modulator. The transformer is a UTC type LS30 which has a frequency response that is flat from 7 cps to 50 kc; therefore, little degradation of the square wave arises from the transformer coupling. The part of the chassis which contains the square wave generator is electrostatically shielded from the part containing the noise source and amplifier, to suppress interference between the noise and the square wave which could arise from stray pick up.

<u>Balanced Modulator</u>--A balanced ring modulator is used to heterodyne the noise output of the amplifier with a carrier frequency of 3.5 kc. Both the noise signal and the carrier are transformer coupled into the modulator to insure that no d-c level is introduced by either input to the modulator. The output of the modulator is directly coupled to the low pass filter so that the frequency of the output noise can extend on the low end of the range to d-c.

The diodes used in the modulator are type 1N646. Four diodes were selected to have the same forward resistance. Precision 500 ohm swamping resistors are connected in series with three of the diodes and a variable resistance is in series with the fourth diode to provide a balance control. The carrier is introduced into the modulator between the center tap of the input transformer and the center tap of the potentiometer that serves as the output load. By adjusting the potentiometer, the carrier can be balanced out to get approximately 70 db of isolation between the input carrier and the carrier present at the output.

The modulator's operation is similar to the operation of a double pole, double throw switch which is being switched at the carrier frequency. This switching action causes the output of the modulator to contain each noise frequency plus the carrier frequency, and each noise frequency minus the carrier frequency. The net result is that of shifting the noise frequency spectrum so that in the new spectrum 0 frequency occurs where a frequency of 3.5 kc was in the original spectrum. This makes the low frequency end of the new spectrum suitable to be used as a low frequency noise source.

Filtering and Output Amplification--After the noise is heterodyned in the ring modulator, it is only necessary to filter and amplify the heterodyned noise to obtain a noise output with the desired characteristics. The filtering is done with a conventional m-derived low pass filter with matching half sections. A plot of the combined frequency response of the modulator and filter is shown in Figure 2. The data for the plot was obtained by introducing as signal to the modulator the output of an audio oscillator and varying the frequency in equal steps from 200 cps below

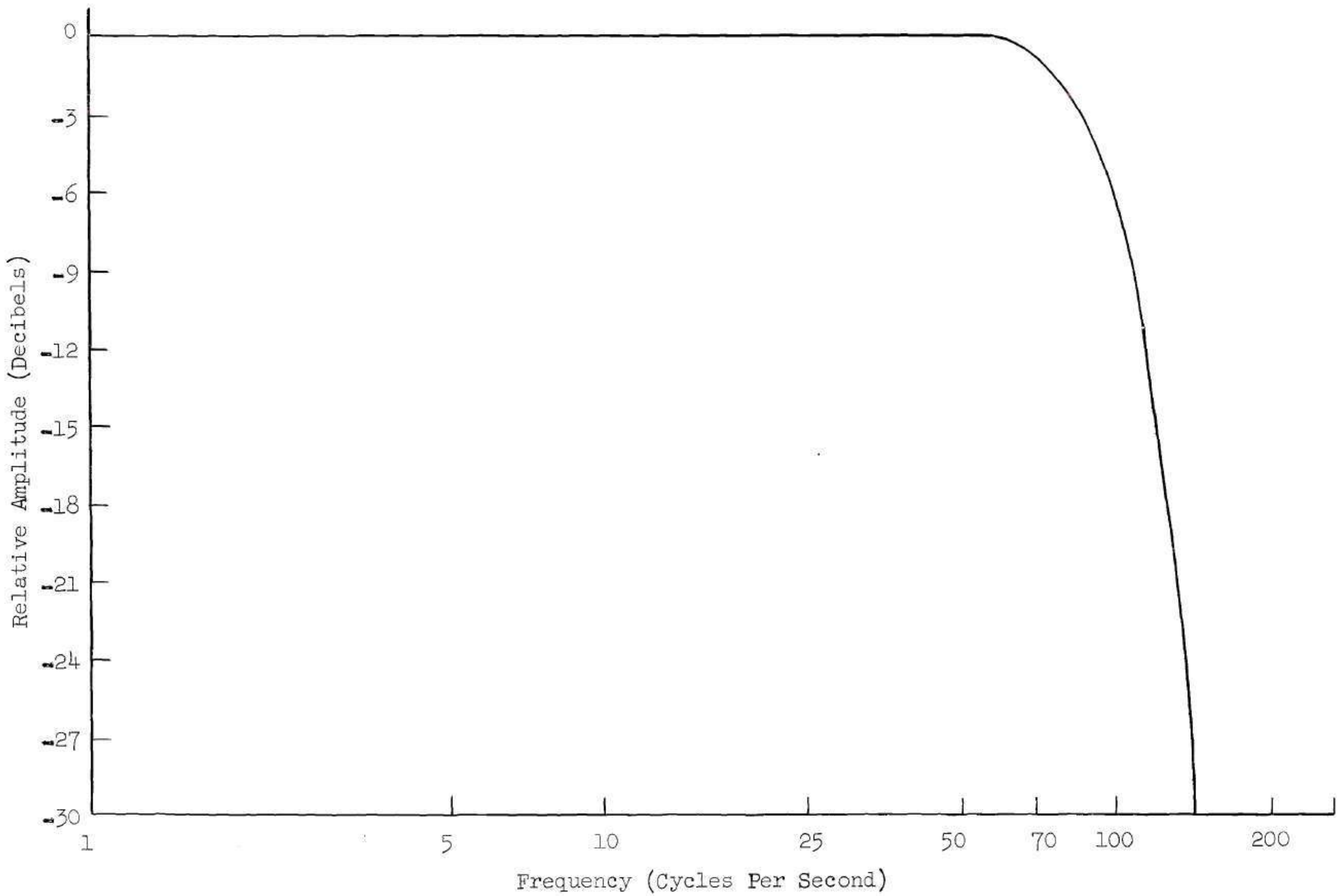

Figure 2. Combined Modulator and Low Pass Filter Response.

the center frequency of the modulator to 200 cps above the center frequency. The output of the low pass filter was then recorded. The frequency response of the combined system is flat to 70 cps.

The output d-c operational amplifier is an original Georgia Tech design and is identical to the amplifier used in the AGC circuit. The schematic diagram for the operational amplifiers is found in Figure 13 in Appendix B. These amplifiers employ controlled internal positive feedback in a directly coupled amplifier to obtain the high gain necessary for them to be used as operational amplifiers. The open loop gain of each amplifier is approximately 20,000. This gain may be adjusted by the procedure described in the operating instructions for the generator given in Appendix A. The limits on the input and output voltages for which these amplifiers can be used are ±50 volts.

The output operational amplifier is connected to have a loop gain which can be varied from 1 to 50 by varying the negative feedback resistor, R_{50}. A balance control R_{37} is provided so that with no input to the amplifier, the output can be set to zero. A 0-center d-c voltmeter is provided in the output circuit of the operational amplifier for this adjustment. The output stage of the operational amplifier is a cathode follower, so the output impedance is in the order of 500 ohms. A sample of the output noise from the generator is shown in Figure 3.

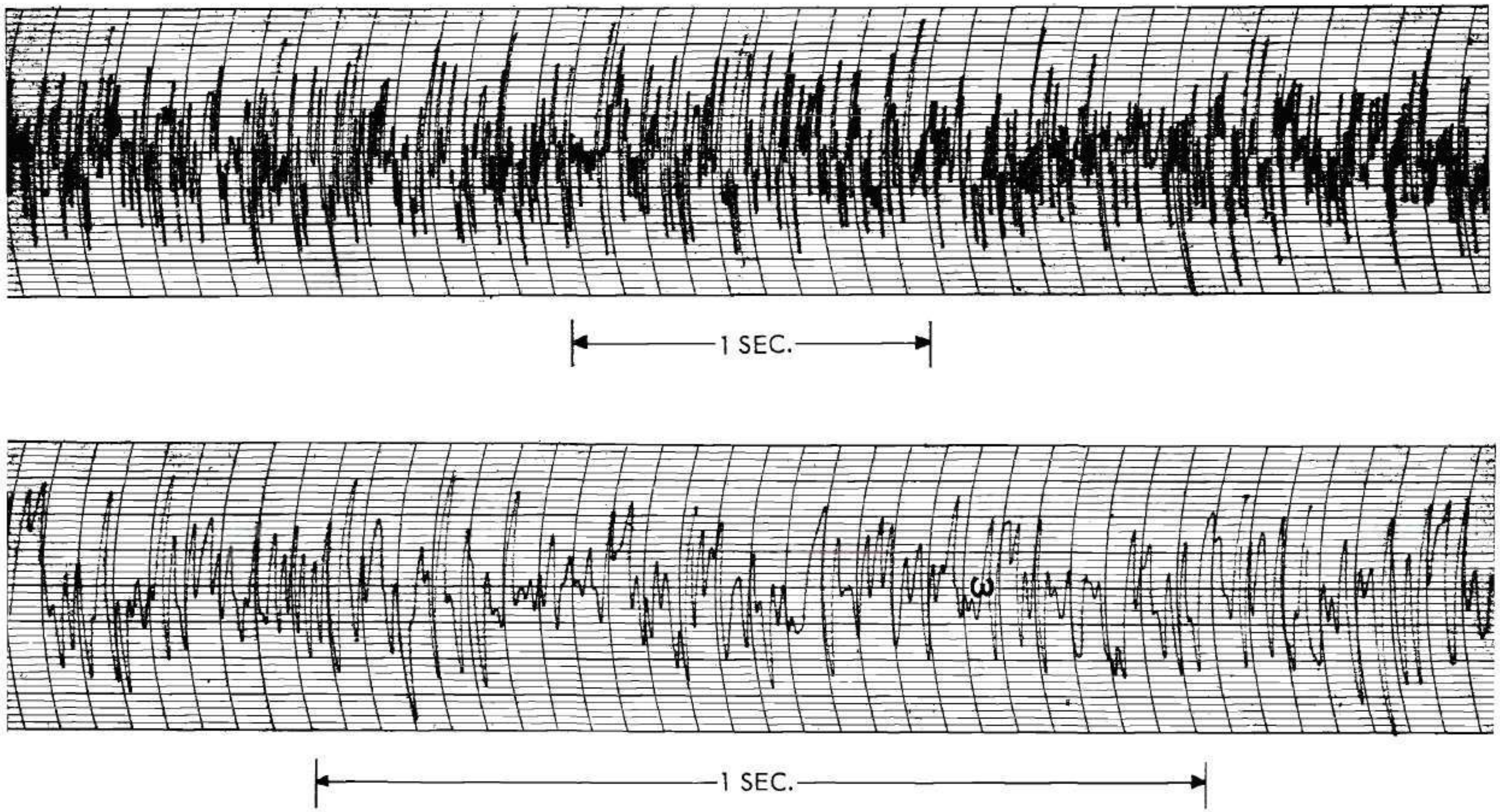

Figure 3. Noise Sample From Generator.

CHAPTER III

RANDOM SIGNAL CONCEPTS

Introduction--Since it is impossible to predict the output voltage of the generator at any time, and likewise impossible to predict the output voltage of the generator for any later time knowing the voltage at one time, the noise output can be considered a random process. A random (or stochastic) process can only be investigated using the notions of probability and statistics. Therefore, to properly test the output of a random noise generator it is necessary to employ various concepts of random variables, probability and statistics. It is the purpose of this chapter to list and define the concepts which will be used in the experimental determination of the characteristics of the noise output. With this purpose in mind, the mathematical definitions will be restricted to apply to "physically realizable" classes of functions. Discussion in this chapter will be restricted to the minimum requirements for the understanding of the experimental tests on the noise. For anyone wanting to devote more time to careful study of random signal analysis there are several excellent texts on the subject (11) (12).

Random Processes--In discussing random processes it is first necessary to understand the concept of an ensemble (also called a set). In general, an ensemble is a collection of objects or entities or time functions such that one is able to determine of any object whether or not it is a part of the collection (13). Let us assume that there are a great number of noise generators all turned on at the same time. The noise outputs ${}^{1}x(t)$,

${}^{2}x(t)$, --- ${}^{k}x(t)$ are then observed. This collection of time functions composes an ensemble. Although mathematically an ensemble is defined in a more general way, here the definition is restricted to countable sets of functions.

It is now possible to define a random process in terms of the ensemble concept. A random (or stochastic) process is usually defined as an ensemble of time functions $\left\{{}^{k}x(t)\right\}$, $-\infty < t < \infty$, $k = 1, 2, 3$ ---, such that the ensemble can be characterized through statistical properties (13). Under this definition, random processes may be subdivided into three classes, namely, stationary processes, non-stationary processes, and ergodic processes.

To be able to categorize random processes into these three classes one must first define ensemble average and time average. Let $F\left[{}^{k}x(t_1)\right]$ be a definite function of ${}^{k}x(t_1)$ at a fixed time $t = t_1$. At this fixed time for a given F, the average of $F\left[{}^{k}x(t_1)\right]$ can be computed over the ensemble. This average may be shown as

$$\tilde{F}\left[x(t_1)\right] = \lim_{N \to \infty} \frac{\sum_{k=1}^{N} F\left[{}^{k}x(t_1)\right]}{N} \tag{2}$$

where $\tilde{F}\left[x(t_1)\right]$ is independent of the ensemble records k since k has been averaged out, but is a function of the particular time t_1 chosen. A time average is taken for a particular record from the ensemble and is

$$\bar{F}\left[{}^{k_1}(x)\right] = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} F\left[{}^{k_1}x(t)\right] dt \tag{3}$$

where in this case $\bar{F}\left[{}^{k_1}(x)\right]$ is independent of time, but is a function of the particular record k_1 being used.

If for a random process it happens that averaging over a large number of records in the ensemble at a time t_1 gives the same results as an average at another time t_2, and if the statistical properties associated with the random process are the same for the ensemble at any time, then this process is said to be stationary. In terms of a function F of the ensemble, this means that for every possible F

$$\tilde{F}\left[x(t_1)\right] = \tilde{F}\left[x(t_1 + t)\right] \text{ for all } t \qquad (4)$$

It is, however, still necessary to take ensemble averages for making statistical measurements of stationary processes. All processes which do not meet the requirements for being stationary are classed as non-stationary processes.

If in addition to being a stationary process, each record of the ensemble of the random process is statistically equivalent to every other record, and the ensemble averages over a large number of records at fixed times may be replaced by corresponding time averages on a single representative record of the ensemble, then this stationary random process is said to be ergodic. This means that, given an ergodic process, a sequence of samples drawn from a single long record can be taken as an ensemble and this new ensemble is statistically equivalent to the original ensemble. Therefore, for every possible function F

$$\tilde{F}\left[x(t)\right] = \bar{F}\left[{}^{k}(x)\right] \text{ for all } k \text{ and } t \qquad (5)$$

The immediate importance of the ergodic hypothesis is evident since it is not feasible to consider an ensemble of noise generators as must be done when a process is either non-stationary or only stationary and not ergodic. Therefore, a time series for a single generator must be used, and an ergodic process must be assumed to justify this course of action.

Autocorrelation Functions and Power Spectral Density--The correlation of a process is used to determine how much dependence one set or time series has on another set or time series. If the correlation is taken between a set at one time and the same set at a different time, the autocorrelation is found. The autocorrelation function in general for any random process $\left\{{}^{k}x(t)\right\}$ at times t_1 and t_2 is written

$$\phi(t_1,t_2) = \lim_{N \to \infty} \frac{\sum\limits_{k=1}^{N} {}^{k}x(t_1)\,{}^{k}x(t_2)}{N} \qquad (6)$$

If the random process is stationary, the general expression for the autocorrelation function can be simplified to be

$$\Gamma(\tau) = \lim_{N \to \infty} \frac{\sum\limits_{k=1}^{N} {}^{k}x(0)\,{}^{k}x(\tau)}{N} \qquad (7)$$

because the stationary process is invariant with respect to time translations. For ergodic processes the autocorrelation function can be derived by a time average

$$R(\tau) = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} {}^{k_1}x(t)\,{}^{k_1}x(t + \tau)\, dt \qquad (8)$$

where a single representative record ${}^{k_1}x(t)$ is used and the answer is independent of k.

There are power spectral density functions defined for non-stationary and non-ergodic processes (13), but for the purpose of this study only the definition of the power spectral density for an ergodic process is necessary. This is defined as

$$G(\omega) = \lim_{T \to \infty} \frac{1}{2T} \left| \int_{-T}^{T} {}^{k_1}x(t)\ e^{-j\omega t}\ dt \right|^2 \tag{9}$$

where ${}^{k_1}x(t)$ is a single record from the ensemble. It can be seen that the function called the power spectral density gives the distribution in frequency of the power of the noise or random process being analysed.

J. S. Bendat (13) shows how the autocorrelation function and the power spectral density can be related by the Fourier Transform. By recognizing that the autocorrelation function is an even function one can express the relation as

$$G(\omega) = \frac{2}{\pi} \int_{0}^{\infty} R(\tau)\ \cos\ \omega\tau d\tau \tag{10}$$

$$R(\tau) = \int_{0}^{\infty} G(\omega)\ \cos\ \omega\tau d\tau \tag{11}$$

One other important operation on a given power spectral density $G_o(\omega)$ of a stationary process, is that of passing it through a linear device that is described by an impedance function $Z(\omega)$ (14). The output of the device will again be a stationary random process and the spectral density will be

$$G_1(\omega) = \left| Z(\omega) \right|^2 G_o(\omega) \qquad (12)$$

<u>Probability Density and Distribution</u>--A real-valued function $x(s)$ defined on a sample space of points s will be called a random variable if for every real number c, the set of points s for which $x(s) \leqq c$ is one of the class of admissable sets for which a probability is defined. If the real random variable is considered to range in x along the real line $(-\infty < x < +\infty)$ and X is a point on the real line, then the function of X whose value is the probability, $P(x \leqq X)$, that the random variable x is less than or equal to X is called the probability distribution (15) function of x. If the variable x is continuous and the probability distribution function is continuous and differentiable then a probability density function $p(x)$ can be defined as the derivative of the probability distribution function.

$$p(x) = \frac{dP(x \leqq X)}{dX} \qquad (13)$$

$$P(x \leqq X) = \int_{-\infty}^{X} p(x)dx \qquad (14)$$

To be meaningful $p(x)$ cannot be negative or imaginary. Also, for the value of x to lie between $-\infty$ and ∞

$$\int_{-\infty}^{\infty} p(x)dx = 1 \qquad (15)$$

and from the non negative character of $p(x)$ and equation (15), the values of $P(x \leqq X)$ must range only from 0 to 1.

Also of importance in discussing probability densities and distributions are the nth moment of the distribution and the nth central moment of the distribution (16). The average of x^n is defined as the nth moment and is written

$$m_n = \widetilde{x^n} = \int_{-\infty}^{\infty} x^n \, p(x)dx \tag{16}$$

It is important to note that m_1 is nothing but the ordinary arithmetic mean and m_2 is the mean square. Frequently it is convenient to analyse data with the average subtracted from all values. The corresponding averages are called central moments and are defined as

$$\mu_n = \widetilde{(x - m_1)^n} = \int_{-\infty}^{\infty} (x - m_1)^n \, p(x)dx \tag{17}$$

The most significant of the central moments is μ_2 which is defined as the variance of the distribution. It can be easily shown that

$$\mu_2 = m_2 - m_1{}^2 = \widetilde{x^2} - (\tilde{x})^2 \tag{18}$$

The square root of the variance is called the standard deviation σ, and is written

$$\sigma = (\mu_2)^{1/2} = \sqrt{\widetilde{x^2} - (\tilde{x})^2} \tag{19}$$

One of the most important distributions in the study of noise is the normal or Gaussian distribution in which the probability density is proportional to the exponential of a negative quadratic function of the variable. The general form for the density for one variable is

$$p(x) = \frac{1}{\sigma\sqrt{2\pi}} \, e^{-(x - x_o)^2/2\sigma^2} \tag{20}$$

where

$$m_1 = x_o = \int_{-\infty}^{\infty} xp(x)dx \tag{21}$$

$$m_2 = x_o^2 + \sigma^2 = \int_{-\infty}^{\infty} x^2p(x)dx \tag{22}$$

$$\mu_2 = \sigma^2 = m_2 - m_1^2 \tag{23}$$

This, of course, means that x_o is the mean, σ^2 is the variance, and σ is the standard deviation.

CHAPTER IV

TEST RESULTS

Introduction--The noise output of the generator should first meet the requirements for a stationary process. The noise should be investigated considering the properties required for being an ergodic process. Then the characteristics which must be known of the noise output of the generator are the rms voltage, the mean value of the noise, the autocorrelation function, the power spectral density, and the first probability distribution function.

Several tests were made to determine if the noise output of the generator is stationary. Autocorrelation functions were plotted for two different samples of noise. The results from both samples were almost identical. The rms voltage of the noise was monitored for a period of four hours and the value remained constant. Although this is no rigorous proof that the noise output is stationary, it gives a good indication that the noise can be assumed a stationary process.

When analysing stationary noise from a single generator, it is customary to consider a number of samples of a given time length, but at different times, to compose the individual records of the ensemble. It was found while computing the first probability distribution of the noise from the generator, that for any given voltage reference level, the probability of the noise being above a particular level was the same for all of the records which were tested from this so called ensemble. This would indicate that the statistical properties of each record are equivalent to those of every other record from the ensemble. This gives good evidence that the noise from the generator can be assumed to be an ergodic process.

Mean and RMS Voltage--The rms voltage of the noise was measured with a thermocouple type voltmeter. The maximum rms output voltage was found to be 3 1/2 volts. Also, tests were made to determine how close the mean value of the noise output of the generator is to zero. The noise was integrated in an analog computer integrator for many five minute periods. If the noise being integrated were with a zero mean the integrator output should remain close to zero. It was found that the noise generator could be adjusted so that the mean was as close as .02 volts to zero. Allowing for subsequent drift, the mean was not greater than .04 volts away from a zero mean.

Autocorrelation Function--The autocorrelation function of a 20 second sample of the noise generator output was computed with the Georgia Tech Analog Correlator. This correlator solves the equation

$$R(\tau) = K\int_0^T v(t)v(t + \tau)dt \qquad (24)$$

for discrete values of τ (17). The input signal to the correlator is recorded on two channels of magnetic tape. The heads which pick up what is recorded on these two channels are positioned adjacent to each other. The time delay (τ) in the function is accomplished by varying the tape distance between the two heads. The delay time (τ) between the two recorded signals can be varied from 0 to 1 second in steps of 83 microseconds. A plot of the normalized, computed autocorrelation function is

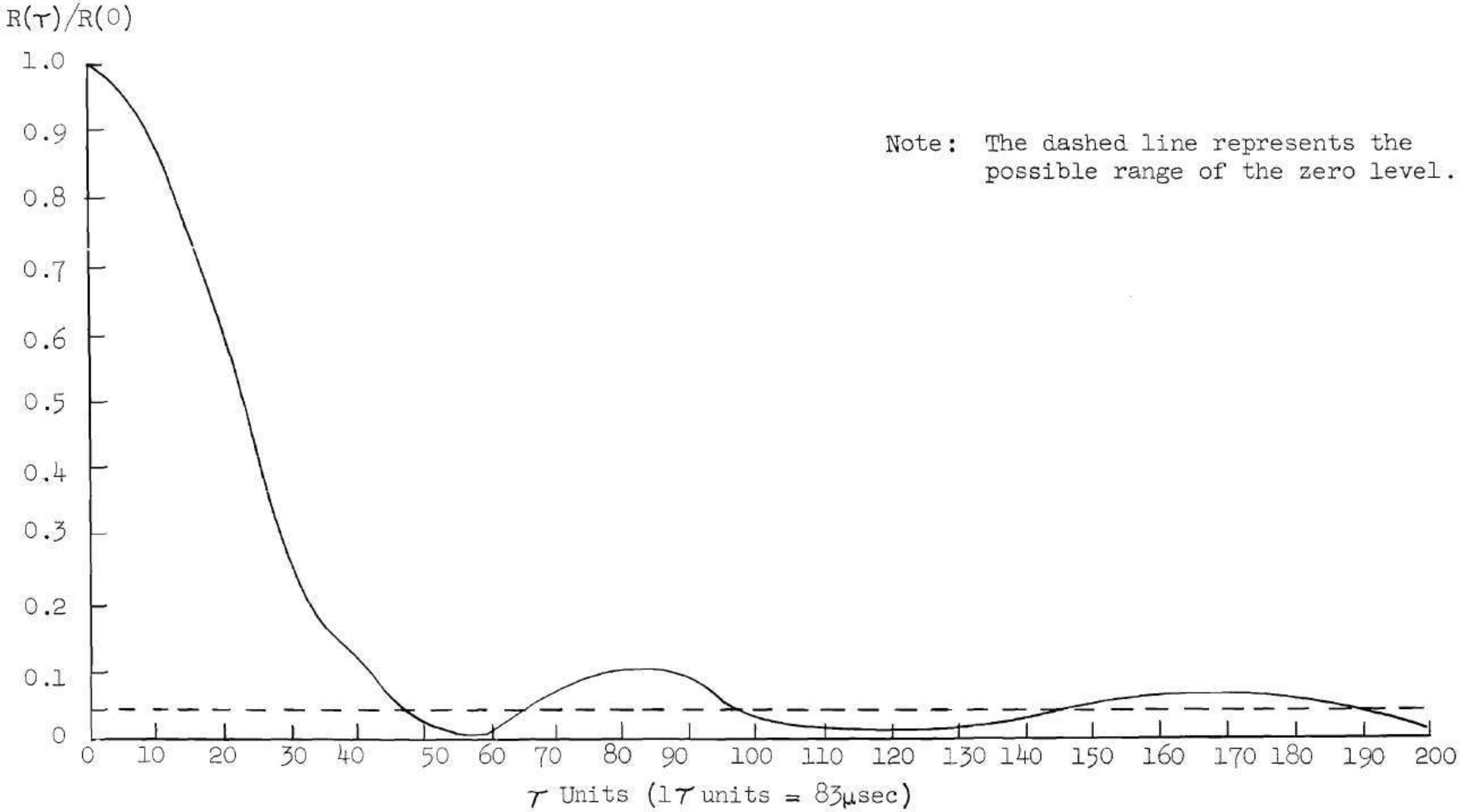

Figure 4. Autocorrelation Curve for 20 second Noise Sample.

found in Figure 4. The function is plotted from $\tau = 0$ to $\tau = 16.6$ milliseconds. The function was computed on the correlator to $\tau = .5$ seconds, but since the function had no variation in level for $\tau > 16$ milliseconds, no further information would be gained from plotting $R(\tau)$ for values of τ greater than those used. It was difficult to determine the exact zero level for the autocorrelation; therefore the function was plotted showing the limits within which the zero level could fall.

The theoretical autocorrelation function for white noise is a delta function at $\tau = 0$. If white noise is passed through an ideal low pass filter which has zero attenuation for $f < f_c$ and infinite attenuation for $f > f_c$, the output of the filter would have an autocorrelation function which would be of the form $\frac{\sin 2\pi f_c}{2\pi f_c}$. Since the characteristic of the filter of this generator is flat to 70 cps and then rounds off, the experimental autocorrelation function should look like a somewhat distorted plot of $\frac{\sin 2\pi f_c}{2\pi f_c}$, as it does.

The autocorrelation function of a random signal gives a good indication of periodicities in the signal. From the computation of the normalized autocorrelation function of a sample output of this generator, periodicities as small as .02 of one normalized unit peak to peak could have been detected. On the normalized correlation function the zero to peak value of any periodic component would be the percentage which the periodic component was of the rms voltage of the entire signal. Since no periodicities appeared in the autocorrelation, it is safe to say that the rms values of any periodic components are at most only 1 percent of the rms value of the noise signal.

Power Spectral Density--The power spectral density of noise from the generator was obtained by taking the Fourier Transform of the autocorrelation function. This was done by programing the Georgia Tech Analog Computer as is shown in Figure 5 to solve the equation

$$G(\omega) = \frac{2}{\pi} \int_0^T R(\tau) \cos \omega\tau d\tau \tag{25}$$

The autocorrelation function $R(\tau)$ was approximated on a function generator to within 1 percent accuracy of the experimental autocorrelation function obtained by the method discussed in the previous section. It was necessary to time scale the program on the analog computer to allow the power spectral density to be computed for frequencies up to 200 cps.

When computing the autocorrelation function, it was difficult to tell exactly where the zero level was on the function; therefore, the power spectral density was calculated for zero at the two different levels shown on the plot of the autocorrelation in Figure 4. One level was taken with the entire plot of the autocorrelation function above the zero level. The other level was taken slightly higher; therefore, letting the function go negative by a small amount. Plots of the power spectral densities obtained for both cases are shown in Figure 6. The actual power spectrum should fall within the limits of the two curves plotted. Although it could not be indicated on the plots of the spectral densities, it was found that for both zero levels there was no power at or above 200 cps.

It is interesting to compare the plots of the power spectral densities with the plot of the combined response of the modulator and low

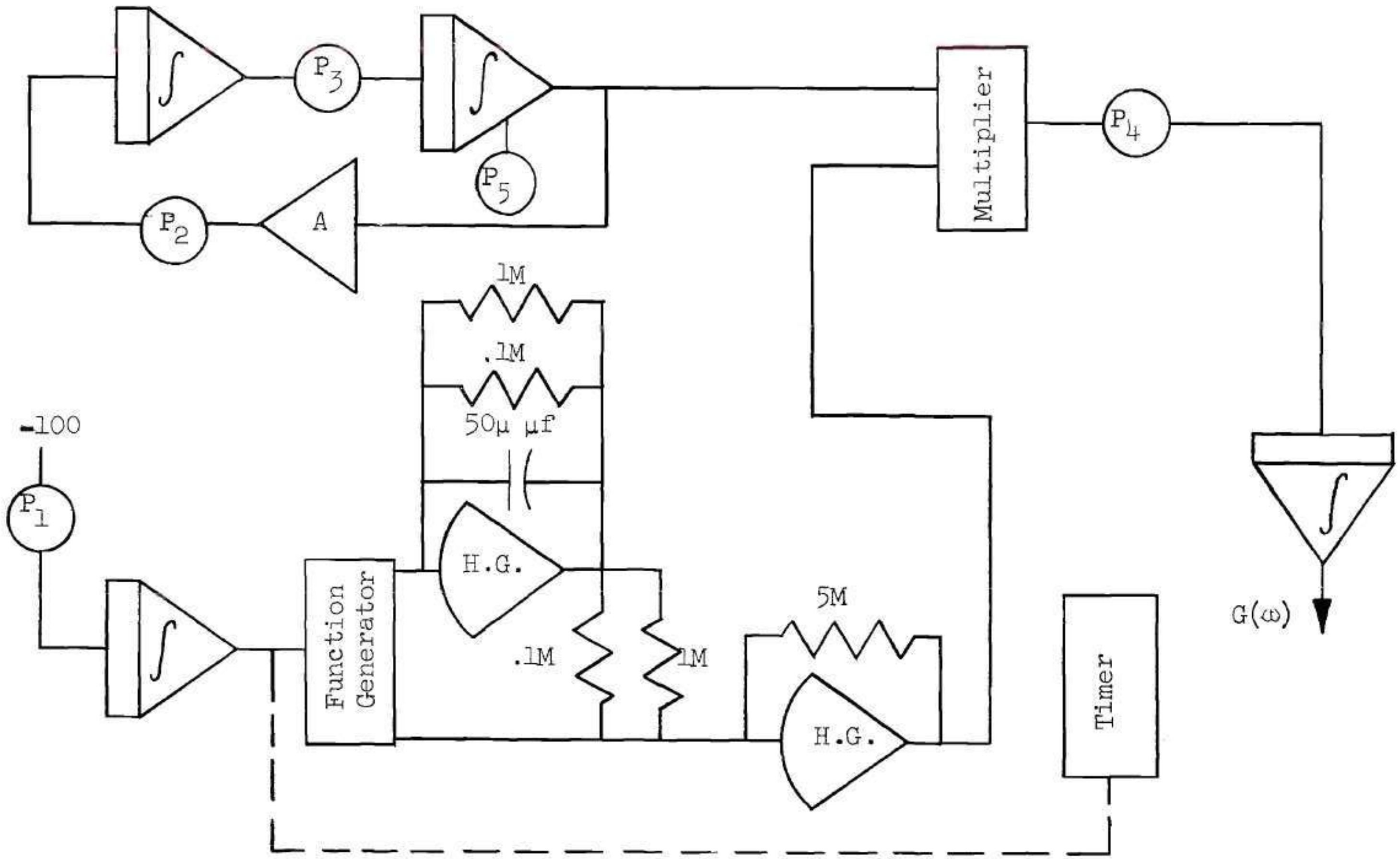

Figure 5. Computer Program for Power Spectral Density.

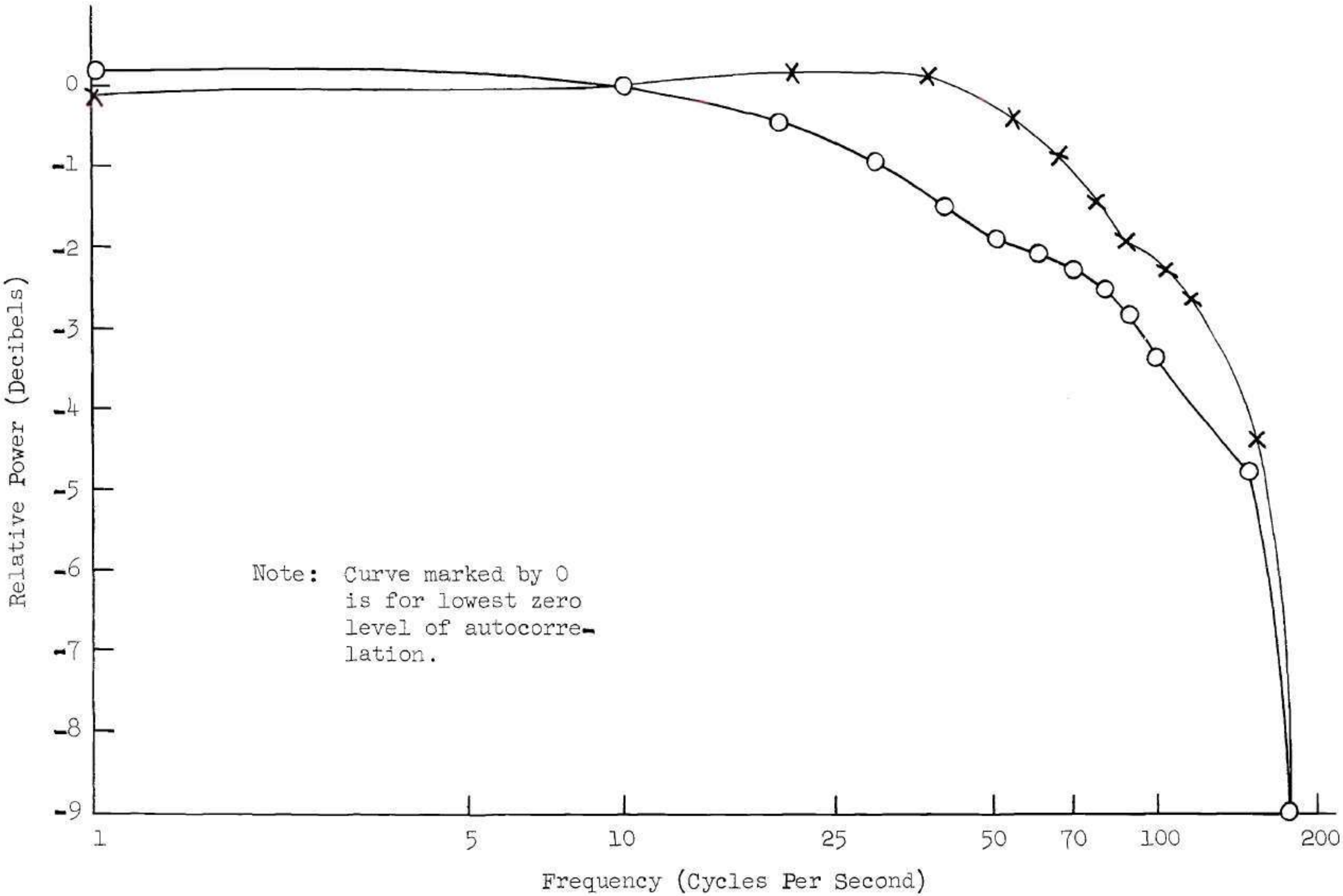

Figure 6. Power Spectral Density for 20 Second Noise Sample.

pass filter shown in Figure 2. As can be seen, the characteristics of the two plots are similar; however, the spectral densities seem to roll off at a lower frequency but then do not decrease as rapidly as the modulator and filter response.

First Probability Distribution--The first probability distribution of the noise from the generator was obtained by programing the Georgia Tech Analog Computer as shown in Figure 7. In the program, the noise is positively biased and amplified so that the noise has large variations around a positive voltage, but never goes negative. This noise is put into a bistable circuit which gives out a constant positive voltage if the noise is above the voltage level set by potentiometer P_3, and almost zero if the noise is below the preset level on P_3. Because the output of the bistable circuit is not exactly zero for noise below the voltage level set on P_3, a summing amplifier is used after the bistable circuit to balance out this small voltage which would cause an error in computation. The signal which is composed of a voltage which is a constant positive value when the noise is above a preset level and zero when the noise is below this level, is integrated for 200 seconds. By properly adjusting the constants of the program, the output of the integrator can be made to be the percentage of time that the noise was above the preset level during the 200 second sample. Since 200 seconds is large compared to the periods of the frequencies containing most of the noise energy, then for the noise signal $x(t)$, what is computed for the 200 second sample is very closely the probability that the voltage $x(t)$ is above some voltage X. In terms of the probability density $p(x)$ the distribution

$$P(x > X) = \int_X^\infty p(x)dx \tag{26}$$

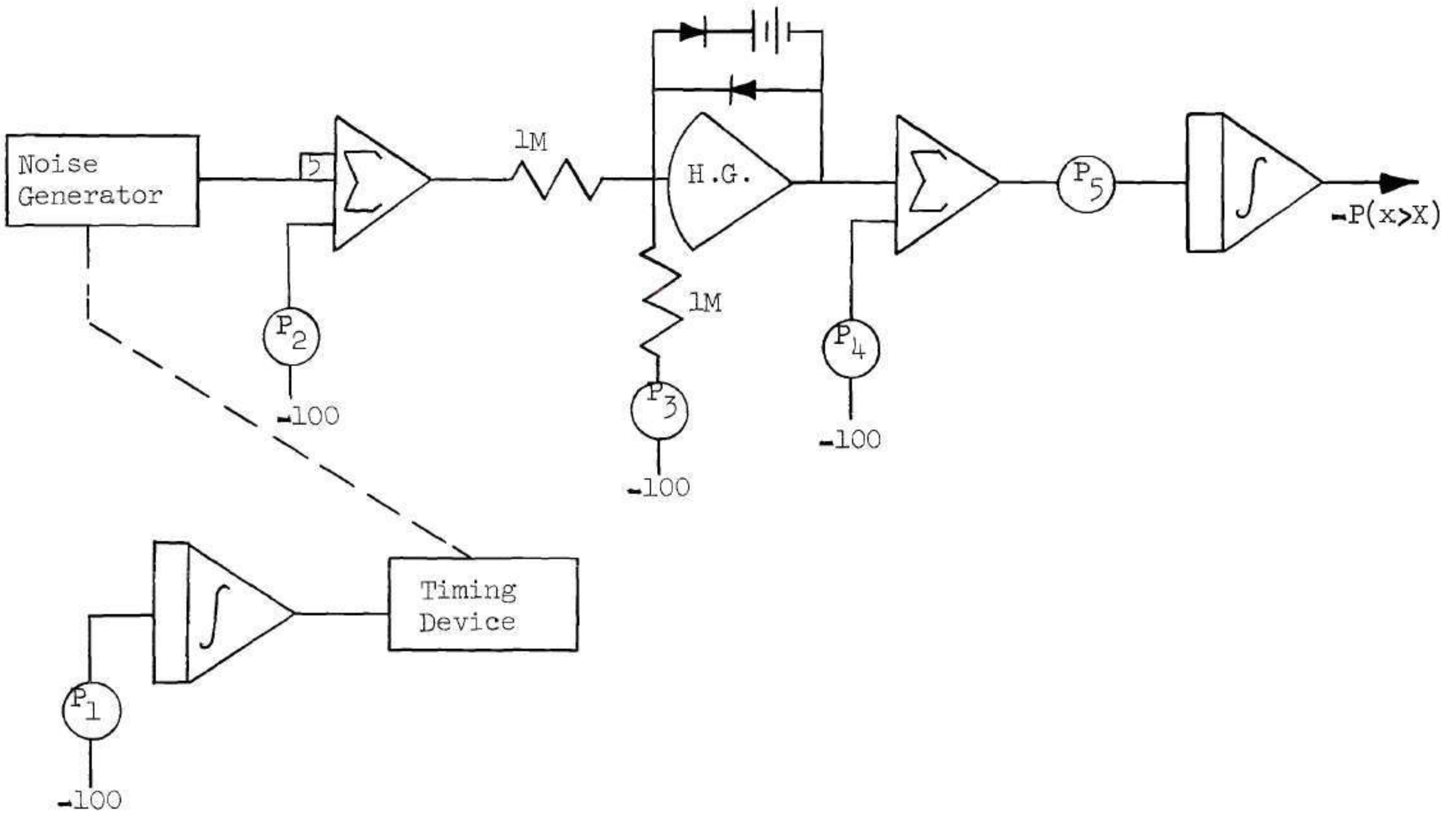

Figure 7. Computer Program for First Probability Distribution.

was computed. The usual form of the distribution function is given by

$$P(x \leq X) = \int_{-\infty}^{X} p(x)dx \tag{27}$$

This distribution is obtained simply by the equation

$$P(x \leq X) = 1 - P(x > X) \tag{28}$$

Since a voltage level was added to the noise to facilitate computation, it is necessary to subtract this value from each level set in the program to obtain the distribution with the proper mean. Also, since $P(x > X)$ was computed, the operation indicated in equation (28) must be performed to get the distribution in the usual form. This distribution was plotted and is shown in Figure 8. It was assumed that this was a normal distribution with a zero mean. The standard deviation (σ) was obtained by finding the voltage values from the curve when $P(X \leq x)$ was .16 and checking this with the voltage when $P(X \leq x)$ was .84. σ was found to be 3.35 volts. A normal curve with a mean of zero and a σ of 3.35 volts was then plotted in the same figure. As can be seen in Figure 8, the theoretical and experimental curves of a normal distribution are almost identical. Therefore, little information could be gained by performing a chi square test on the experimental distribution (18).

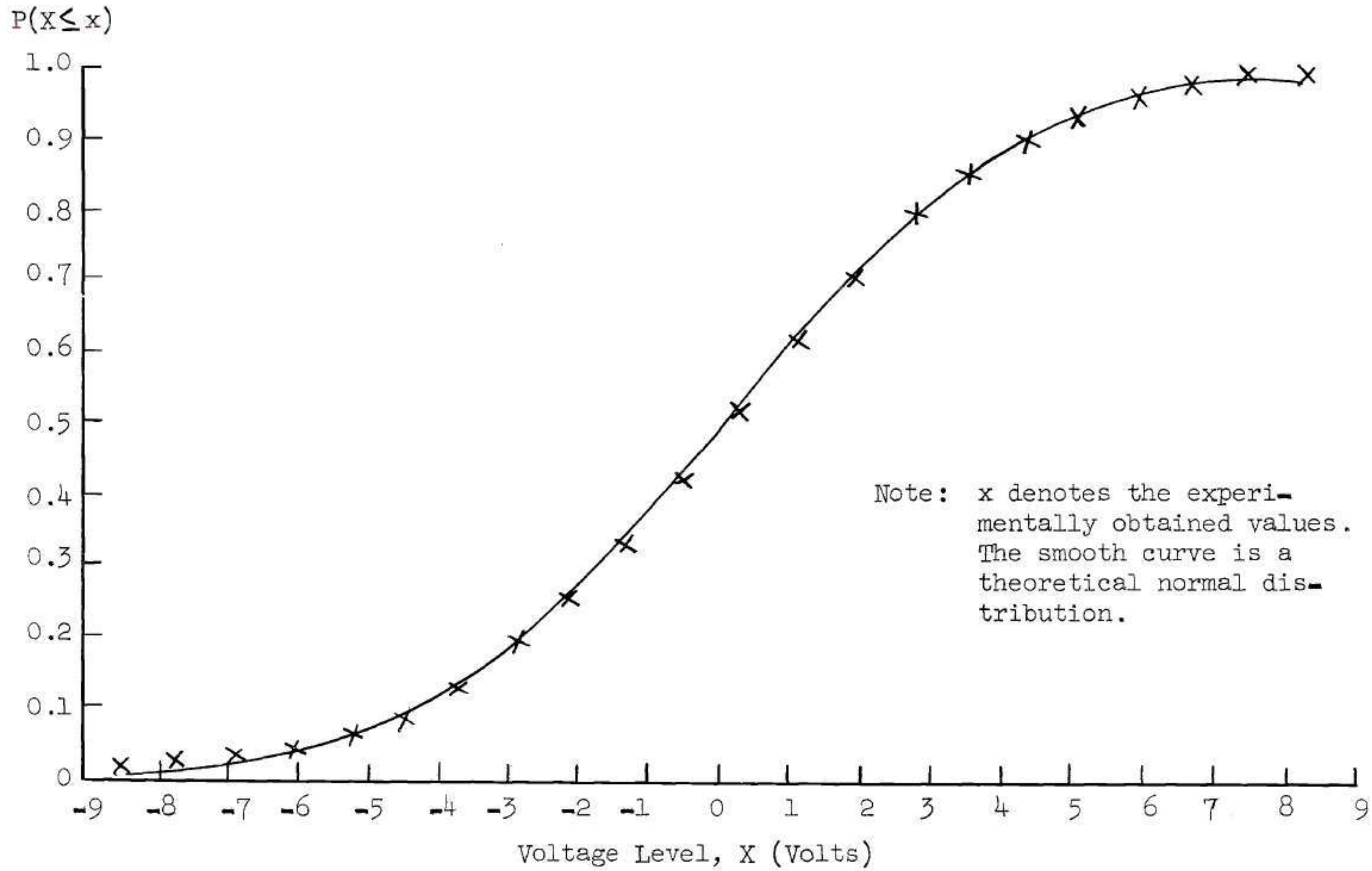

Figure 8. First Probability Distribution of Noise Output.

CHAPTER V

CONCLUSIONS AND RECOMMENDATIONS

From the tests made on the noise output of this generator it is felt that the generator complies with the requirements needed to produce a noise signal for simulation studies on an analog computer. The rms voltage of the noise signal is large enough to eliminate problems encountered by stray pick up. The mean of the noise signal can be set very close to zero. The autocorrelation function, the power spectral density, and the first probability distribution of the noise output of the generator, all are close enough to the theoretically expected functions for the noise from the generator to be considered a random process with a normal amplitude distribution and a power spectral density which is flat within 1 db to 50 cps.

There are several areas in which further investigation into the design and testing of this generator might prove interesting and enlightening. Other than the 6D4, there are several tubes such as the 2D21 and the Burroughs Type 6700 which are commonly used as noise sources. Using one of these tubes as a primary noise source in the generator might alter the power spectral density and the amplitude distribution of the noise output.

It would probably be found that by using a chopper stabilized operational amplifier in the output stage of the generator, that the mean value of the noise could be set closer to zero. Also it might be interesting to try an rms type detector in the automatic gain control circuit rather than the average value one being used now.

For testing the output of the generator, there are methods for obtaining its statistical characteristics other than those used in this thesis. For example, the power spectral density might have been obtained by using a spectrum analyzer which is usually a narrow bandpass filter with a variable center frequency. It would be informative to see how the results using such methods compare with the results obtained in this thesis.

APPENDIX A

APPENDIX A

OPERATING INSTRUCTIONS

The power requirements of the noise generator are:

Filament Supply	- 5 amps at 6.3 volts
Bias Supplies	- 15 milliamps at -300 volts - battery at 90 volts
Plate Supply	- 60 milliamps at +300 volts

All power except that from the 90 volt battery is brought into the chassis through a six pin Jones Plug Connection. The power into the chassis is fused, and switches and pilot lights are provided on the front panel for distributing the power throughout the system. Always allow the filaments to heat for at least 30 seconds before turning on the plate power.

After the generator has warmed up for about 30 minutes the set up procedure can be performed. The only equipment necessary is a good cathode ray oscilloscope and a screw driver. Output terminals are provided on the back of the chassis for all points needed to be monitored in the set up procedure.

First turn R_3 to its limit in the counter-clockwise direction and also adjust R_7 so that the AGC bias meter does not read over nine milliamps. Now the noise output is zero and the modulator can be balanced. First it is necessary to adjust the square wave for symmetry. This can be done by connecting the "Square Wave Out" terminal to the oscilloscope and adjusting R_{42} until both half cycles of the square wave have equal widths.

To balance the modulator, connect the oscilloscope to the terminal marked "Modulator Out" and adjust R_{27} for minimum amplitude of square wave output. The peak-to-peak value should be approximately .2 volts. After the modulator is balanced while the noise output is still zero, the "Balance" control on the front panel can be adjusted so that the "Zero Set" meter reads zero.

Now the AGC bias level and noise output can be set. Turn R_3 clockwise until it is about one quarter of a revolution from its upper limit. Now adjust R_7 until the "AGC Bias" meter reads 8 milliamps. This will put the proper amplitude noise signal into the modulator. The set up is complete except for the arbitrary gain setting of the "Gain" control on the front panel.

For a more accurate adjustment of the mean of the noise output at zero, an additional adjustment can be made. Connect the noise output of the generator to an analog computer integrator. By adjusting R_{26} the integrator output can be made to remain very close to zero and not build up in value. If the range of R_{26} is not enough, slight adjustments of the "Balance" control on the front panel can be made.

The gain of the two operational amplifiers may also be adjusted, however, this adjustment is needed very infrequently. This setting is made by adjusting R_{13} on each amplifier. This must be done, however, when the amplifier is connected in a test chassis. The amplifier is connected with unity gain and an input which can be either plus or minus 30 volts. The gain resistor, R_{13}, is then adjusted so that the voltage on the amplifier marked terminal "no. 5" in Figure 13 remains unchanged as the input is changed from +30 volts to -30 volts.

It is not necessary to perform the entire set up procedure each time the generator is used. The modulator should need balancing only about every two weeks. It is, however, necessary to make the balance and bias adjustments before each operation. No other forms of adjustment or maintenance are foreseen other than the replacement of any faulty vacuum tubes or components.

APPENDIX B

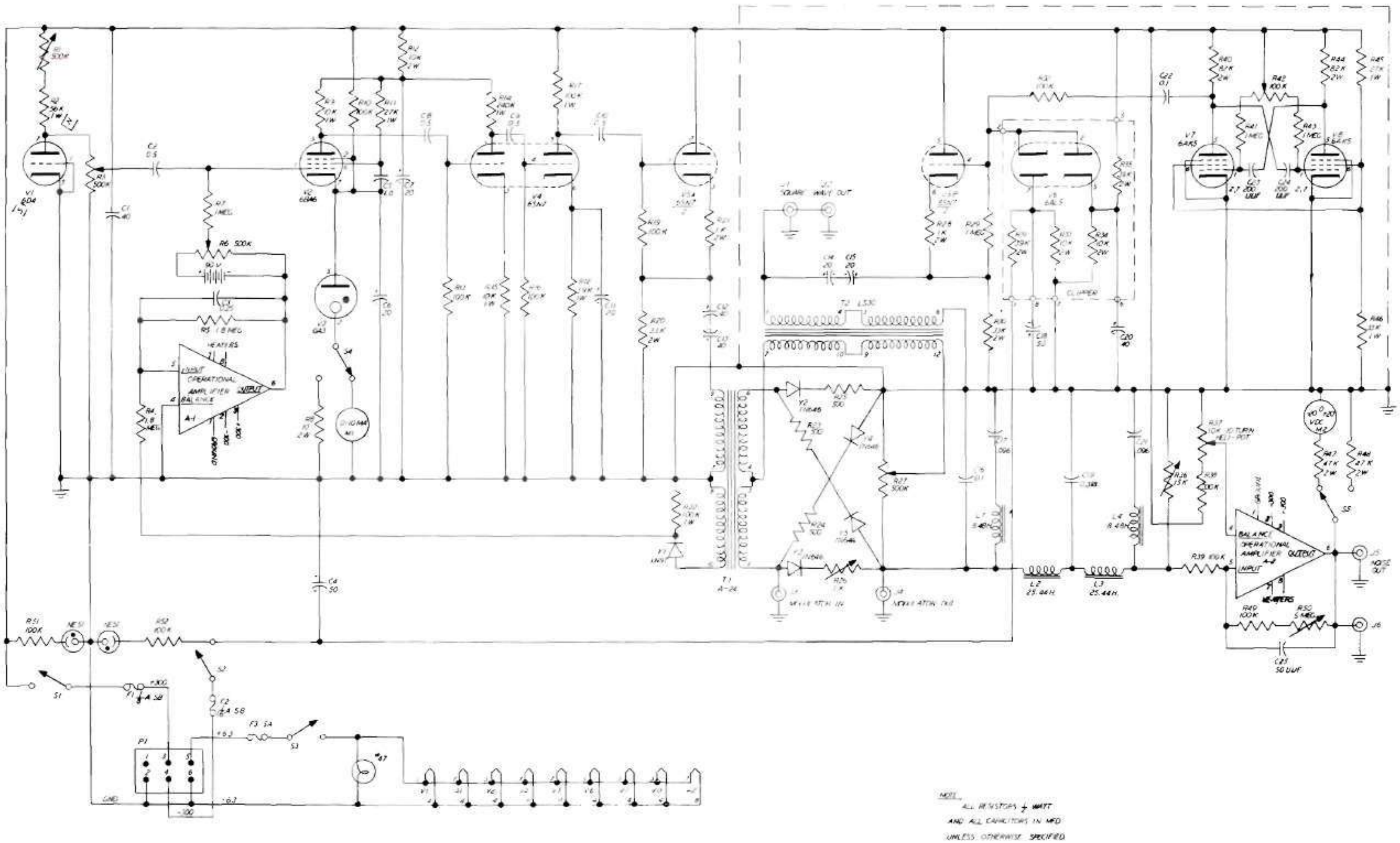

Figure 9. Schematic Diagram of Noise Generator.

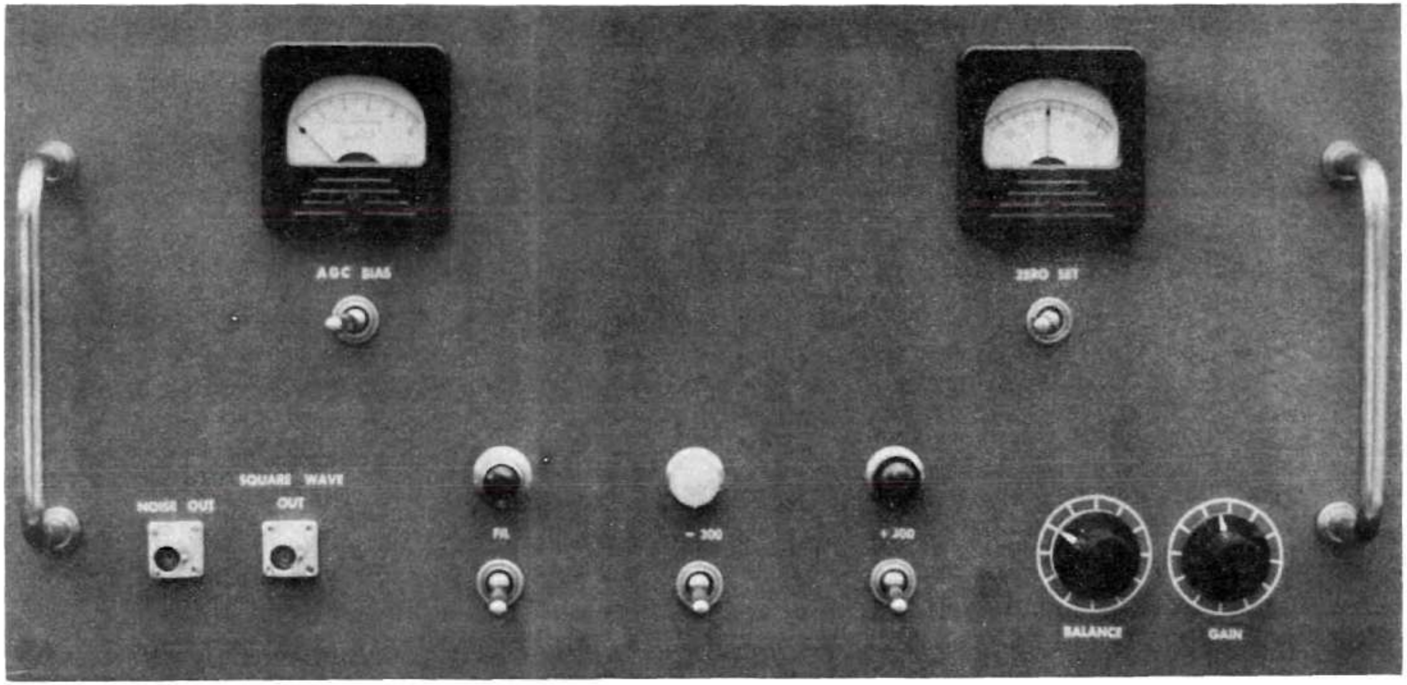

Figure 10. Noise Generator, Front View.

Figure 11. Noise Generator, Top View.

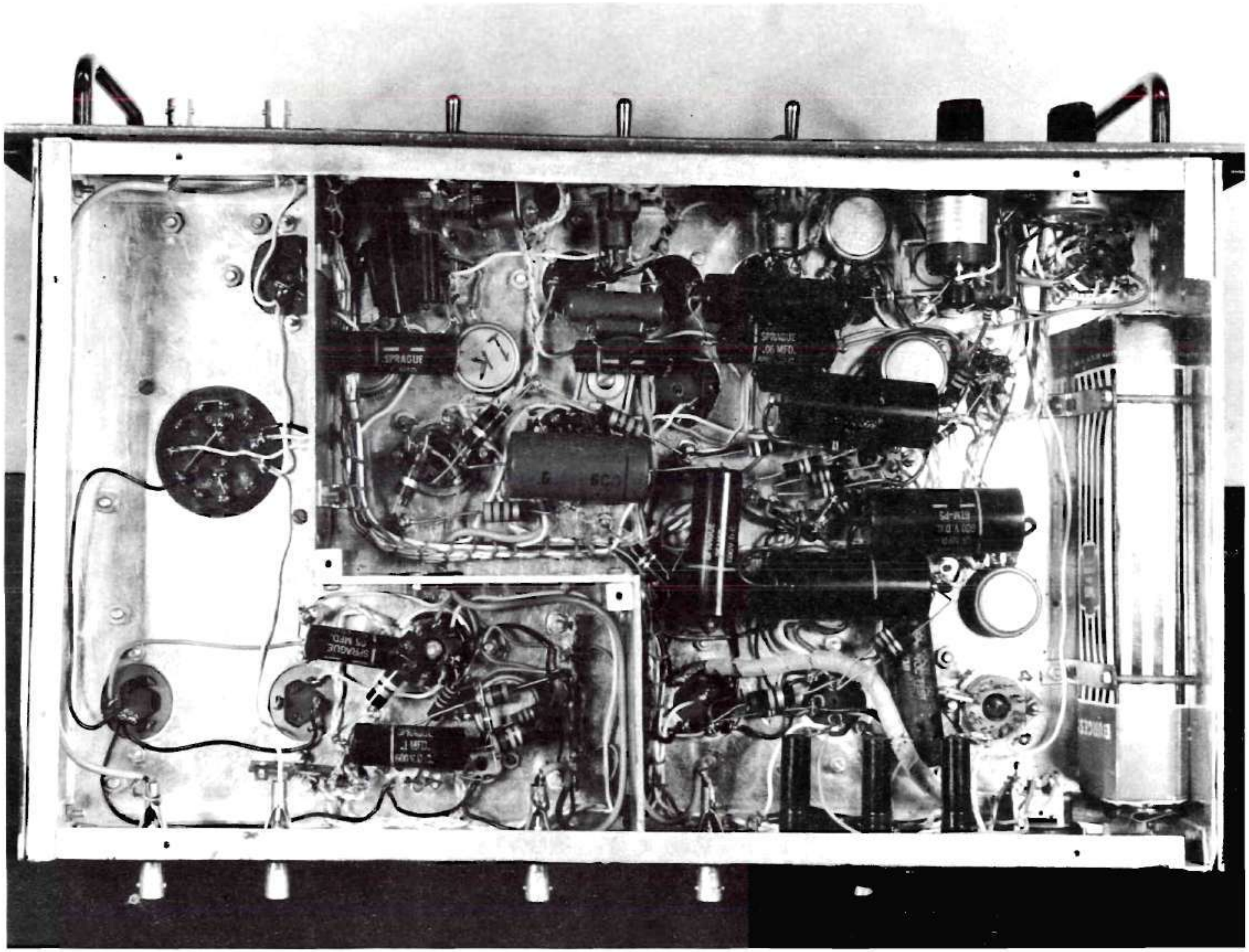

Figure 12. Noise Generator, Bottom View.

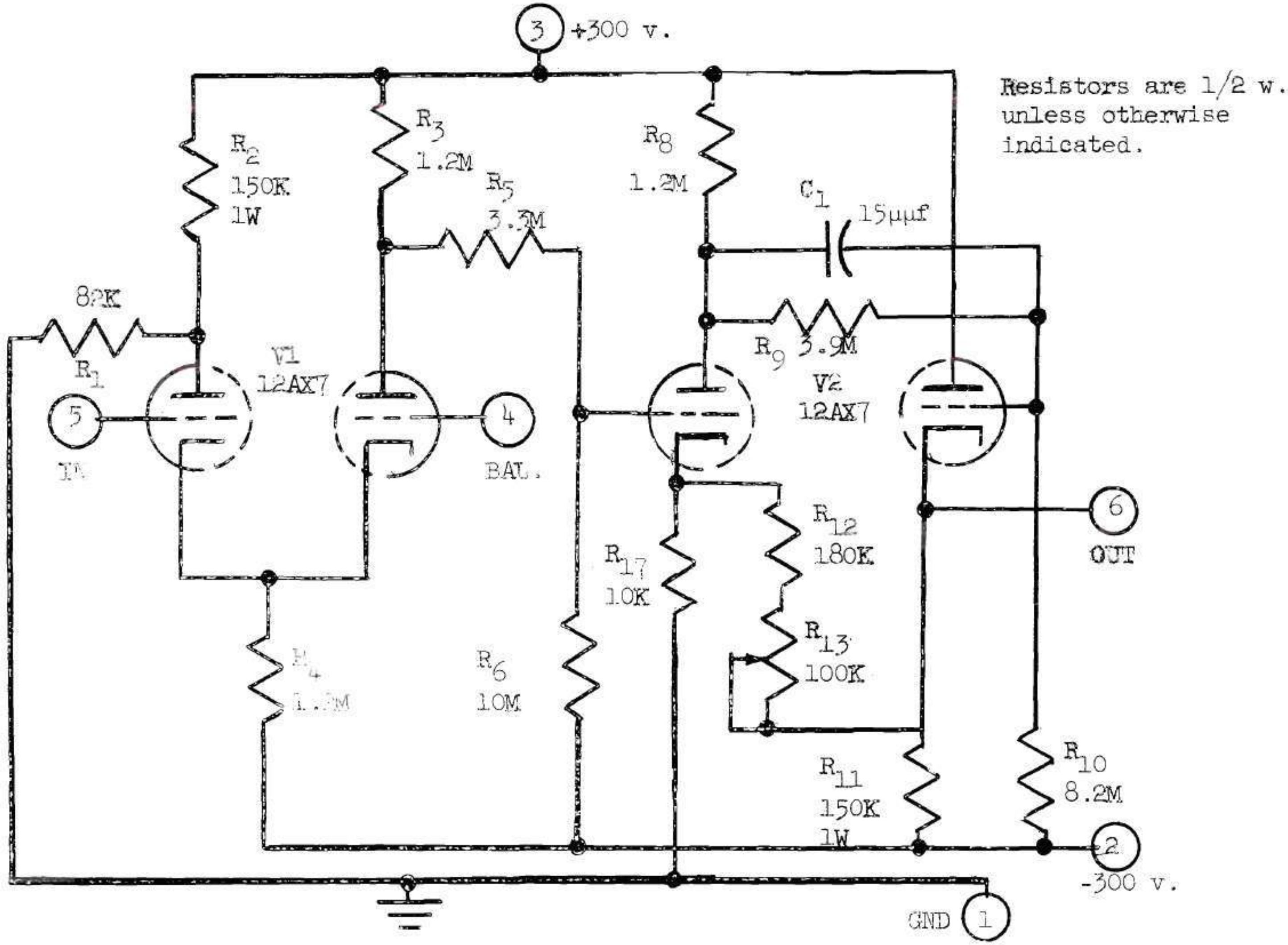

Figure 13. Schematic Diagram of Georgia Tech D-C Amplifier.

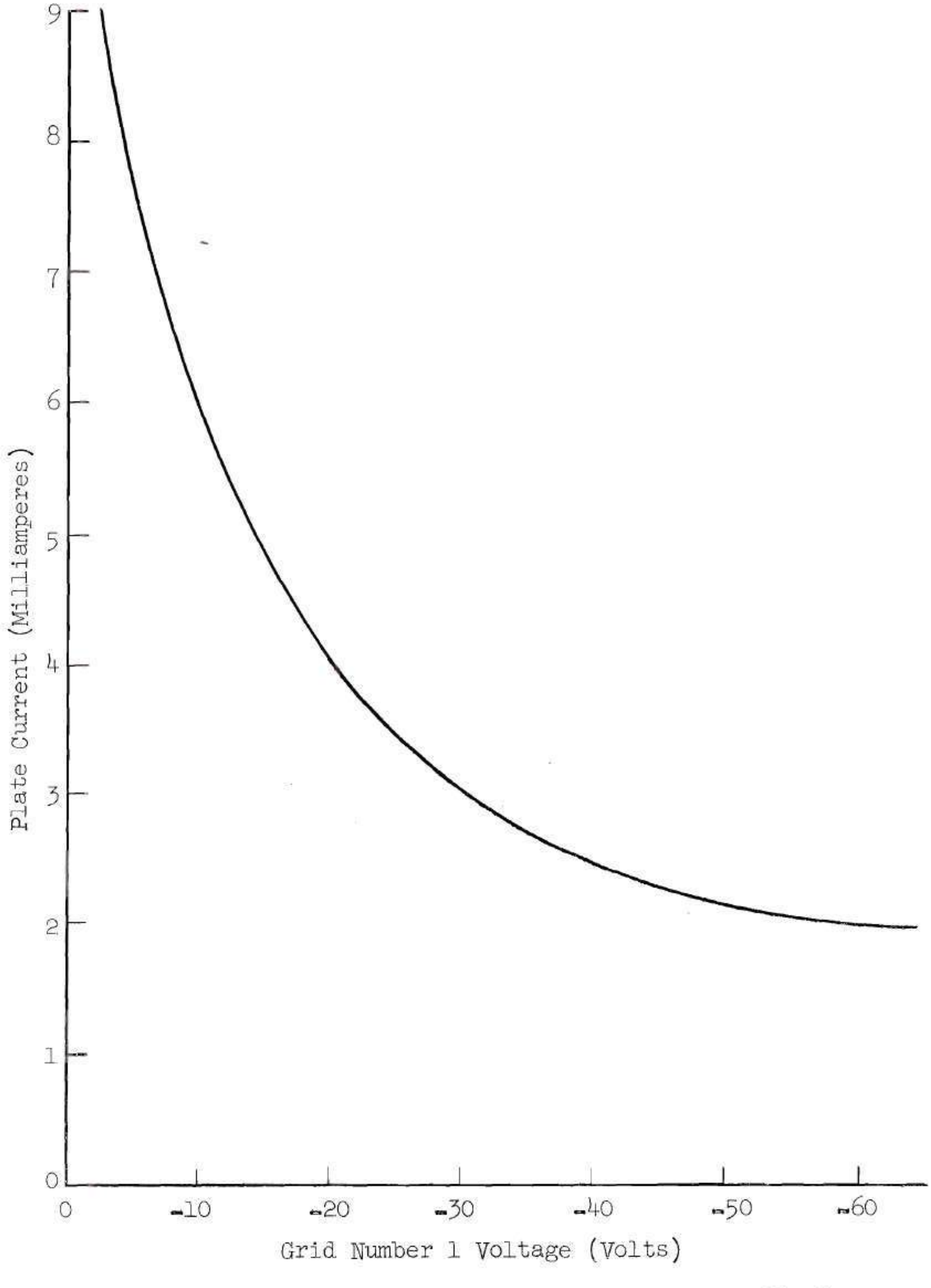

Figure 14. Average Transfer Characteristic (6BA6).

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